Universität zu Köln
Markets for Risk Management

Problem Set #3 Solutions Name <u>SOLUTIONS</u>
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<u>Problem #1</u>

Suppose a corporation has issued zero coupon debt that matures 1 year from now. Management has promised to pay creditors €10 million at that date, provided that the firm is solvent. Corporate assets are currently worth €15 million, and the (annual) standard deviation of the return on these assets is 40%. Assume that the firm will be liquidated 1 year from today, that there is no corporate income taxation, and that the (annual) rate of interest is 4%.

A. What is the risk neutral probability that the firm will be insolvent 1 year from today?

<u>SOLUTION</u>: The risk neutral probability that the firm will be insolvent 1 year from today is determined from the Black-Scholes call option pricing equation. If we characterize the firm's equity as representing a call option in its assets, then this probability corresponds to the probability that the call expires out of the money; i.e., that the firm becomes insolvent. Equivalently, this would correspond to the probability that the "limited liability" put option is profitably exercised. Either way, from the equation, this probability therefore corresponds to 1- $N(d_2)$. Calculating $N(d_2)$ requires computing the value of d_2, and in order to know d_2, we must find the value for d_1:

$d_1 = \dfrac{\ln(S_0/X)+(r+.5\sigma^2)T}{\sigma\sqrt{T}} = \dfrac{\ln(15/10)+(.04+.5(.40)^2)}{.40} =$ 1.3137. Therefore, $d_2 = d_1 - \sigma\sqrt{T}$ = 1.3137 - .40 = 0.9137. From the standard normal probability table, $N(d_2)$ = 81.96%, which means that the risk neutral probability that the firm will be insolvent 1 year from today is 1- $N(d_2)$ = 1 - .8196 = 18.04%.

B. Suppose that after 6 months, the value of the firm's assets has risen from €15 million to €16 million. If this were to occur, what is the risk neutral probability 6 months from today that the firm will default on its debt 1 year from today? Explain the economic logic concerning why the risk neutral insolvency probability has changed.

<u>SOLUTION</u>: The passage of 6 months time makes the date of expiration closer, so this has the effect of making it more likely that the firm is insolvent. However, since the underlying asset is worth more, this has the reverse effect. We compute 1-$N(d_2)$ using these new parameter values:

$d_1 = \dfrac{\ln(S_0/X)+(r+.5\sigma^2)T}{\sigma\sqrt{T}} = \dfrac{\ln(16/10)+(.04+.5(.40)^2.5)}{.40\sqrt{.5}} =$ 1.8738. Therefore, $d_2 = d_1 - \sigma\sqrt{T}$ = 1.8738 - .40(.7071) = 1.5910. From the standard normal probability table, $N(d_2)$ = 94.42%, which means that the risk neutral probability 6 months from today that the firm will default on its debt 1 year from today is 1- $N(d_2)$ = 1 - .9442 = 5.58%.

C. Suppose the firm can reduce the volatility of its assets from 40% to 10% by purchasing an actuarially fair hedge on the value of its assets. Should management authorize such a hedge? Why or why not?

<u>SOLUTION</u>: The answer to this is obvious even without any numerical computation. We know that in the absence of taxes and other costs related to financial distress such as agency costs and bankruptcy costs, that limited liability creates a moral hazard by protecting shareholders from downside risk and allowing them to enjoy gains on the upside. All of the benefits of hedging go to the bondholders at the expense of the shareholders. Therefore, management should not authorize such a hedge.

This insight can also be shown numerically. Without taxes, the value of the firm's equity today is €5,708,623.81. By hedging at an actuarially fair price, equity declines €316,517.17 to €5,392,106.65. Since the value of the firm's assets doesn't change, this means that the firm's debt enjoys a corresponding €316,517.17 increase in value from €9,291,376.19 to €9,607,893.35.

D. Now suppose that the government institutes a 50% tax rate on corporate income, which is defined as terminal asset value minus the sum of debt repayment plus depreciation allowances. Depreciation of €15,000,000 will be claimed by the firm. Under these circumstances, should management authorize such a hedge? Why or why not?

<u>SOLUTION</u>: Solving this problem requires some numerical computation. By allowing corporate income taxation, we have created a tax incentive to hedge, since reducing risk simultaneously reduces the value of shareholders' pre-tax option on the firm, as well as the value of the tax option. Since shareholders are concerning with after-tax value, these effects obviously offset each other.

With taxes, the value of the firm's equity today is €5,488,394.42. By hedging at an actuarially fair price, equity declines to €5,392,106.41. For all practical purposes, this change makes it virtually unlikely that the government will receive any money. However, bonds are worth much more. Since hedging in this case reduces firm value, the optimal decision by management would be to not authorize this hedge.

Problem #2

Suppose that a firm owns assets worth €20,000,000. These assets are financed with publicly traded equity and zero coupon bonds that have a face value of €8,000,000. The standard deviation of the return on the firm's assets is 30%. Assume that this firm will be liquidated one year from today and that the annual rate of interest is 2%.

The firm is considering an investment that costs €3,000,000 and has a (pre-tax) net present value of €1,500,000. If this investment is made, the firm plans to issue additional zero coupon bonds with a face value of €1,500,000. The project will be financed with the proceeds from the bond issue plus cash. However, since the new investment is very risky, if adopted it will increase the overall risk of the firm's assets from a standard deviation of 30% to 50%.

Corporate taxes are assessed at a 35 percent rate whenever taxable income is positive. However, if the firm loses money, then its tax liability is zero dollars; i.e., the government does not provide tax rebates. Taxable income is defined as the difference between the time 1 value of the firm's assets less the sum of the face value of debt plus depreciation allowances. Currently, depreciation allowances total €12,000,000. If this investment is undertaken, then depreciation allowances will total €15,000,000.

A. Should your company undertake this project? Why or why not?

SOLUTION:

In the absence of taxes (i.e., if $\tau = 0\%$), the project would be undertaken since on a pre-tax basis it has a positive net present value. However, we must consider what impact the adoption of this project would have upon the firm's expected tax liability, since this will affect the after-tax value of the firm's shares. Therefore, in order to properly evaluate this project, we must consider what the after-tax value of the firm's shares will be before and after the adoption of the project. This requires calculating the pre-tax value of the firm's equity and the value of the government's tax option.

After-tax value of equity (assuming the investment is not undertaken):

The pre-tax value of the firm's equity (assuming the investment is not undertaken) can be computed by calculating d_1 and d_2, then $N(d_1)$ and $N(d_2)$, and then combining these probability measures with the current value of assets and the current value of safe bonds.

$$d_1 = \frac{\ln(S_0/X)+(r+.5\sigma^2)T}{\sigma\sqrt{T}} = \frac{\ln(\$20 \text{ million}/\$8 \text{ million})+(.02+.5(.09))}{.3} = 3.271.$$

Therefore, $d_2 = d_1 - \sigma\sqrt{T} = 3.271 - .3 = 2.971$. Consequently, $N(d_1) = 99.95\%$, $N(d_2) = 99.85\%$, and the pre-tax value of equity (assuming the investment is not undertaken) is:

$V_0^1(E) = V_0^1(F)N(d_1) - e^{-rT}BN(d_2)$ = €20 million(.9995) – $e^{-.02}$(€8 million)(.9985) = €12,159,331.32.

The value of the government's tax option (assuming the investment is not undertaken) can be computed by calculating d_1 and d_2, then $N(d_1)$ and $N(d_2)$, and then combining these probability measures with the current value of assets and the present value of the sum of sum of the face value of debt plus depreciation allowances.

$$d_1 = \frac{\ln(S_0/X)+(r+.5\sigma^2)T}{\sigma\sqrt{T}} = \frac{\ln(\$20 \text{ million}/\$20 \text{ million})+(.02+.5(.09))}{.3} = 0.2167.$$

Therefore, $d_2 = d_1 - \sigma\sqrt{T} = 0.2167 - .3 = -0.0833$. Consequently, $N(d_1) = 58.58\%$, $N(d_2) = 46.68\%$, and the value of the tax option (assuming the investment is not undertaken) is:

$$V_0^1(T) = \tau\left\{V_0^1(F)N(d_1) - e^{-rT}(B+dep)N(d_2)\right\} =$$
$$.35\{€20 \text{ million}(.5858) - e^{-.02}(€20 \text{ million})(.4668)\} = €897{,}510.92.$$

Combining results, the after-tax value of the firm's shares is:

$$V_0^1(E) - V_0^1(T) = €12{,}159{,}331.32 - €897{,}510.92 = €11{,}261{,}820.41.$$

<u>After-tax value of equity (assuming the investment is undertaken):</u>

The pre-tax value of the firm's equity (assuming the investment <u>is</u> undertaken) can be computed by calculating d_1 and d_2, then $N(d_1)$ and $N(d_2)$, and then combining these probability measures with the current value of assets and the current value of safe bonds.

$$d_1 = \frac{\ln(S_0/X)+(r+.5\sigma^2)T}{\sigma\sqrt{T}} = \frac{\ln(21.5 \text{ million}/9.5 \text{ million})+(.02+.5(.25))}{.5} = 1.9235.$$

Therefore, $d_2 = d_1 - \sigma\sqrt{T} = 1.9235 - .5 = 1.4235$. Consequently, $N(d_1) = 97.28\%$, $N(d_2) = 92.27\%$, and the pre-tax value of equity (assuming the investment <u>is</u> undertaken) is:

$$V_0^1(E) = V_0^1(F)N(d_1) - e^{-rT}BN(d_2) = €21.5 \text{ million}(.9728) - e^{-.02}(€9.5 \text{ million})(.9227)$$
$$= €12{,}322{,}896.90.$$

The value of the government's tax option (assuming the investment <u>is</u> undertaken) can be computed by calculating d_1 and d_2, then $N(d_1)$ and $N(d_2)$, and then combining these probability measures with the current value of assets and the present value of the sum of sum of the face value of debt plus depreciation allowances.

$$d_1 = \frac{\ln(S_0/X)+(r+.5\sigma^2)T}{\sigma\sqrt{T}} = \frac{\ln(21.5 \text{ million}/24.5 \text{ million})+(.02+.5(.25))}{.5} = 0.0288$$

Therefore, $d_2 = d_1 - \sigma\sqrt{T} = 0.0288 - .5 = -0.4712$. Consequently, $N(d_1) = 51.15\%$, $N(d_2) = 31.87\%$, and the value of the tax option (assuming the investment <u>is</u> undertaken) is:

$$V_0^1(T) = \tau\left\{V_0^1(F)N(d_1) - e^{-rT}(B + dep)N(d_2)\right\} =$$

$$.35\{€21.5 \text{ million}(.5115) - e^{.02}(€24.5 \text{ million})(.3187)\} = €1{,}169{,}797.22.$$

Combining results, the after-tax value of the firm's shares (assuming the investment is undertaken) is:

$$V_0^1(E) - V_0^1(T) = €12{,}322{,}896.90 - €1{,}169{,}797.22 = €11{,}153{,}099.68.$$

Comparing the after-tax values of the firm's shares pre- and post-investment, we find that on an after-tax basis, the adoption of this project would reduce shareholder value by €108,720.73, so it should not be undertaken.

Bondholders will prefer that the investment not be undertaken, since the effect of dramatically increasing the risk of the firm's assets is to make the bonds less secure. Since the total NPV is €1,500,000 and we know that the shareholder and government shares of this NPV are -€108,720.73 and €272,286.31 respectively, this means that bond values increase by €1,336,434.42. However, since the face value of bonds is increasing by €1,500,000, the effect of adopting the project would be to cause bondholders to bear more default risk, thus increasing the bond yield from 2.03% to 3.52%. Consequently the adoption of the project reduces the welfare of the existing bondholders.

B. Now suppose Congress amends the corporate tax code so that the investment described above would provide a 5% investment tax credit of €150,000. Would such a change in the tax code affect the firm's decision to invest in this project? Why or why not?

SOLUTION:

Since the NPV to shareholders is -€108,720.73 without an investment tax credit, the addition of an investment tax credit would cause the investment to increase shareholder value by €150,000-€108,720.73 = €41,279.27. Therefore this change in the tax code will cause my company to accept rather than reject this project.

C. Now suppose that instead of introducing an investment tax credit, Congress lowers the corporate income tax rate from 35 percent to 15 percent. If this happens, should your company undertake this project? Why or why not?

SOLUTION:

Applying the same solution procedure as in Problem 1, we find that the adoption of this project will cause shareholder value to increase by €46,871.45. Therefore my company should undertake this project if Congress lowers the corporate income tax rate from 35 percent to 15 percent.

D. At what tax rate is the firm indifferent about making the investment? Justify your answer.

SOLUTION: Using solver, we find that the tax rate that does this is 21.0249%.

Problem #3

Suppose that shareholders are contemplating making an investment today (at t=0) and are considering different financing alternatives. The payoffs on this investment occur one period from today (at t=1). At t=1, only two states of the world (loss and no loss) may occur with equal probabilities. If the loss occurs, the investment will be worth €2,000, and if there is no loss, then the firm will be worth €4,000. However, the firm has an option to rebuild the asset at a cost of €1,600 should a loss occur. Assume that shareholders are risk neutral, the interest rate is zero and bankruptcy is costless.

A. What is the net present value of rebuilding the asset?

SOLUTION: Given this problem's assumptions, the NPV equals the expected value of the net cash flow benefit from rebuilding. Since a rebuilding cost of €1,600 occurs when the firm suffers a €2,000 loss, this means that the NPV is therefore equal to .5(€2,000-€1,600) = €200. We can also compute NPV by comparing the value of an all equity firm before and after the investment is made. Let Π = income before loss, L_s = state contingent loss, and I_s = state contingent building cost. Furthermore, let $V(E_{r,s})$ represent the state contingent value of equity when there is rebuilding, and $V(E_{n,s})$ represent the state contingent value of equity when there is no rebuilding. Then NPV = E[V($E_{r,s}$)] – E[V($E_{n,s}$)], where

$$E[V(E_{r,s})] - E[V(E_{n,s})] = \sum_{s=1}^{n} p_s V(E_{r,s}) - \sum_{s=1}^{n} p_s V(E_{n,s}).$$

Therefore,

$$\sum_{s=1}^{n} p_s V(E_{r,s}) = \sum_{s=1}^{n} p_s (\Pi - I(s)) = .5(4{,}000\text{-}0) + .5(4{,}000\text{-}1{,}600) = €3{,}200, \text{ and}$$

$$\sum_{s=1}^{n} p_s V(E_{n,s}) = \sum_{s=1}^{n} p_s (\Pi - L(s)) = .5(4{,}000\text{-}0) + .5(4{,}000\text{-}1000) = €3{,}000; \text{ consequently,}$$

$$\text{NPV} = E[V(E_{r,s})] - E[V(E_{n,s})] = €3{,}200 - €3{,}000 = €200.$$

B. Suppose the firm is all equity financed. Will shareholders rebuild the asset in the event of a loss? Why or why not?

SOLUTION: As we showed in part A above, by rebuilding the asset we make ourselves €200 richer, so we will rebuild the asset in the event of a loss.

C. Suppose that as an alternative to equity financing, shareholders can issue zero coupon bonds. If the promised payment on the bonds equals €2,000, will shareholders rebuild the asset in the event of a loss? Why or why not?

SOLUTION: We know that underinvestment can occur when there is default risk. However, with a €2,000 promised payment on the bonds, there is no possibility of default, irrespective of whether the loss occurs. Therefore, bonds are worth €2,000, and the value of equity is €3,600-€2,000 = €1,200 if the asset is rebuilt, compared with

€3,000-€2,000 = €1,000 if the asset is not rebuilt. Consequently, shareholders will rebuild the asset in the event of a loss, because to do so makes them €200 richer.

D. Suppose shareholders issue zero coupon bonds and promise to repay €3,000. With this type of financing arrangement, will shareholders rebuild the asset in the event of a loss? Why or why not?

SOLUTION: Now, we introduce the possibility of default, so things may change, since the benefits of the investment will accrue to bondholders rather than shareholders. Specifically, let $E[V(B_{n,s})]$ be the value of the firm's bonds if there is underinvestment, and $E[V(B_{r,s})]$ equal the value of the firm's bonds if rebuilding occurs:

$$E[V(B_{n,s})]=\sum_{s=1}^{n} p_s\{Min(B,\Pi-L(s))\} = .5(\text{Min}(€3{,}000,\ €4{,}000) + .5(\text{Min}(€3{,}000,\ €4{,}000-€2{,}000)$$

= .5(€3,000) + .5(€2,000) = €2,500, and

$$E[V(B_{r,s})]=\sum_{s=1}^{n} p_s\{Min(B,\Pi-I(s))\} = .5(\text{Min}(€3{,}000,\ €4{,}000) + .5(\text{Min}(€3{,}000,\ €4{,}000-€1{,}600)$$

= .5(€3,000) + .5(€2,400) = €2,700.

Since the total value of the firm is €3,000 when there is underinvestment and €3,200 otherwise, this implies that the value of the firm's shares is €500 no matter what happens. However, shareholders are not indifferent about rebuilding. In the present case, the prospect of default creates a moral hazard. Shareholders would like to convince bondholders that they will rebuild in the event of a loss, collect higher proceeds from issuing bonds, and renege on their promise. Since bondholders are rational and recognize the incentives for shareholders to employ such a tactic, they will assume the worst; viz., that shareholders will not rebuild. Consequently, bonds will be priced at €2,500, which guarantees that shareholders will not rebuild (since bondholders get all the benefits and shareholders do not have anything to gain from rebuilding).

E. Suppose that instead of issuing zero coupon bonds with a promised repayment of €3,000, shareholders decide to issue zero coupon bonds with a promised repayment of €2,600 and purchase an actuarially fair insurance policy with a deductible of €1,400. With this type of financing arrangement, will shareholders rebuild the asset in the event of a loss? Why or why not?

SOLUTION: By altering the face value of debt and purchasing insurance, the firm makes bonds completely safe, so the market value of bonds must be €2,600. We know this occurs because the net cash flow in the loss state will be equal to €4,000 less the rebuilding cost of €1,600 plus the payment from the insurer of €200, or €2,600. However, by introducing insurance, shareholder incentives change. Since the payoff on the firm's equity will be €0 when the loss occurs and €1,400 otherwise, this means that shares will be worth €700 if rebuilding occurs and €500 otherwise. In other words, shareholders appropriate the full net present value from rebuilding. Therefore, shareholders will rebuild the asset.

F. Now suppose that the insurance is not actuarially fair; specifically, all policies have 50% premium loadings. If shareholders issue zero coupon bonds with a promised repayment of €2,800 and purchase an actuarially unfair insurance policy with a deductible of €1,200, will they rebuild the asset in the event of a loss? Why or why not?

SOLUTION: The following table summarizes the payoffs that will occur under this scenario:

State	p_s	Π	L_s	I_s	Max(I_s - d,0)	Π^*	D_s^r	S_s^r
No Loss	50%	€4,000.00	€0.00	€0.00	€0.00	€4,000.00	€2,800.00	€1,200.00
Loss	50%	€4,000.00	€2,000.00	€1,600.00	€400.00	€2,800.00	€2,800.00	€0.00
value now		€4,000.00	€1,000.00	€800.00	€200.00	€3,400.00	€2,800.00	€600.00

This financial arrangement guarantees that the firm does not default on its debt. Thus, bonds are worth €2,800. Since the actuarially fair price for insurance is €200, this means that the net present value enjoyed by shareholders is reduced by the amount of the transaction costs, which in this case is .5(200) = €100. However, since shareholders are richer from rebuilding than they are if they don't rebuild, they will optimally decide to rebuild the asset.

G. Why do the net present values computed in parts E and F differ from each other?

SOLUTION: The €100 difference in net present value (in part E, NPV was €200, and only €100 in part F) is a direct result of the fact that the firm must pay an insurer €100 in transactions costs under the scenario described in part F.